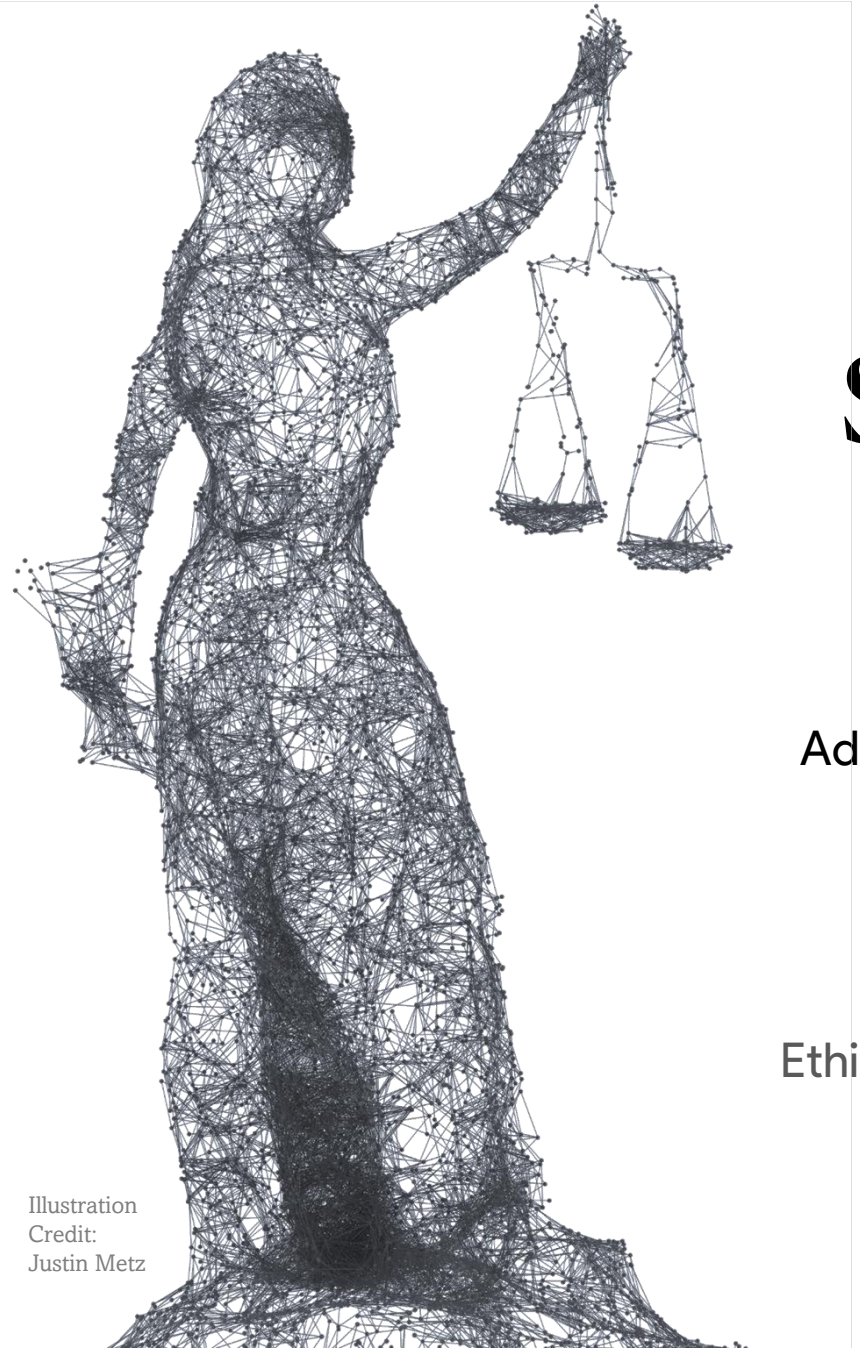

Illustration Credit: Justin Metz

# Structural Group Unfairness

## Measurement and Mitigation by means of the Effective Resistance


Adrian Arnaiz-Rodriguez

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Georgina Curto

Nuria Oliver

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Ethics of Data Analytics and AI - University of Notre Dame – Guest Lecture

20 February 2024

# Main takeaways

Social Networks are very useful to analyze social dynamics or make predictions about people and their relationships

Specific [Arnaiz-Rodriguez, Curto and Oliver 2024]:

Your position on a **social network** defines the access and control of information you have

↓

Social Capital

It is not easy to **quantify it** and intricated to **align the** literature in sociology and graph theory/fairness

↓

Quantitative measures of social capital sociology-aligned based on graph theory

There are some individual and group inequalities and not trivial to fix

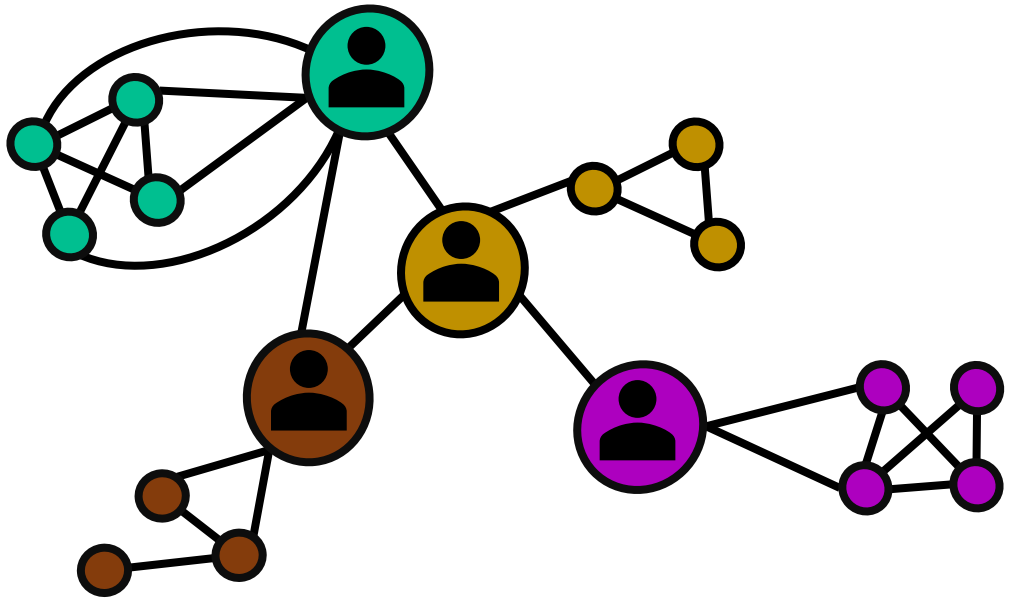
↓

Fix it using an edge augmentation strategy based on the weak**est** links

Arnaiz-Rodriguez, Adrian, Georgina Curto, and Nuria Oliver.
"Structural Group Unfairness: Measurement and Mitigation by means of the Effective Resistance." *Submitted*. (2024)

# Social Networks

Social Networks (or Graphs) are defined by people and interactions between them

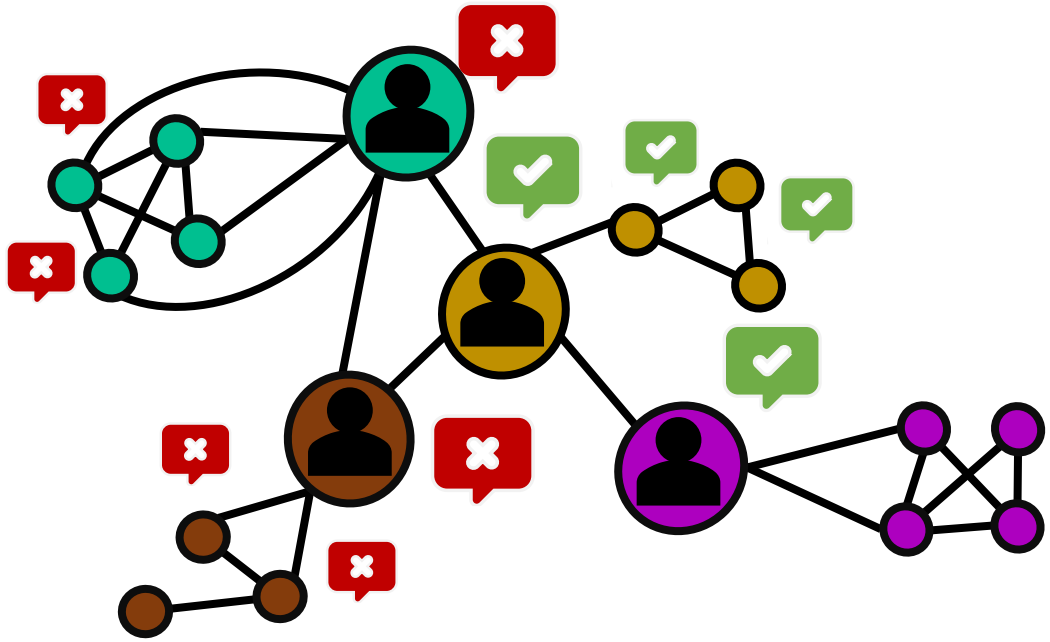


Formally, networks are defined as a set **nodes** (people) connected by **links** (interactions)

# Social Networks

Formally defined, we can analyze different social aspects:

- **Predictions** about **people**

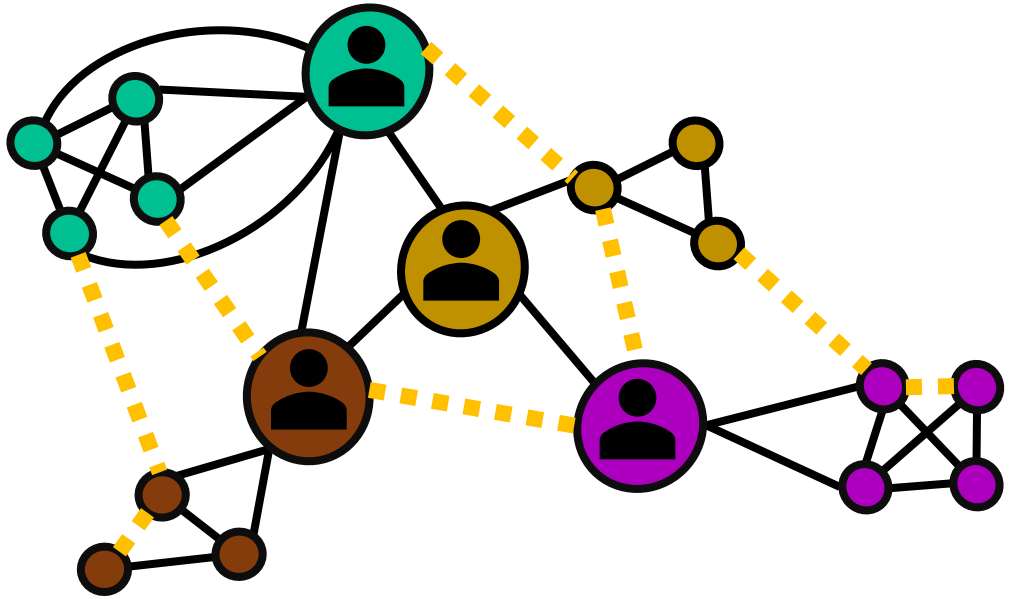


**Risk:** if there are segregated or discriminated communities might lead to biased decisions

# Social Networks

Formally defined, we can analyze different social aspects:

- **Predictions** about people or about their **relations** (Recommenders Systems)

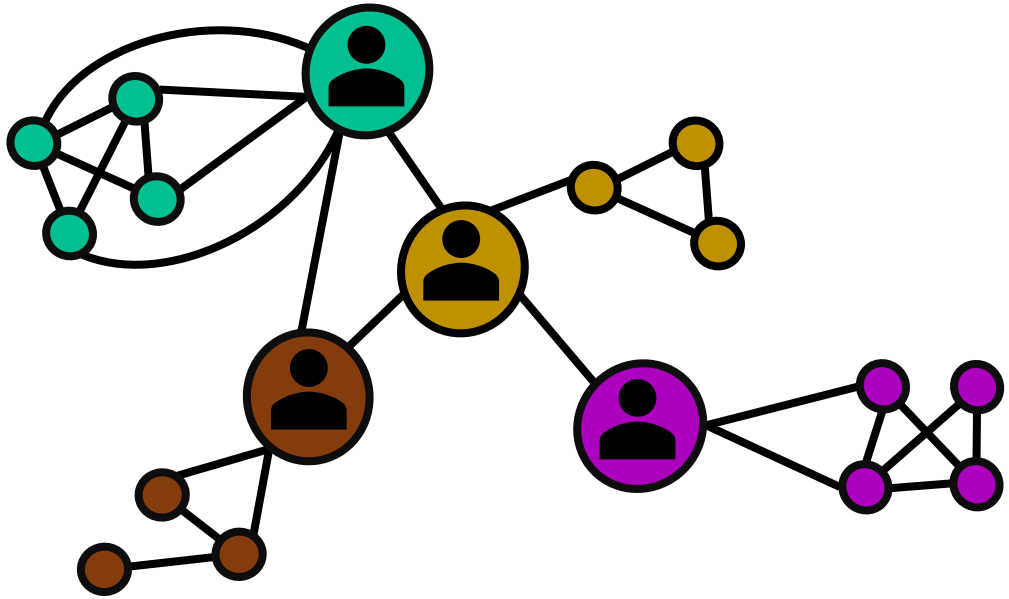


**Risk**: it might lead to polarization or discrimination (disparate information access) due to homophilic behavior

# Social Networks

Formally defined, we can analyze different social aspects:

- **Predictions** about people or about their relations
- **Analyze importance** of people, the relations, the overall **flow of information**, the social **dynamics**,...



Source: Google

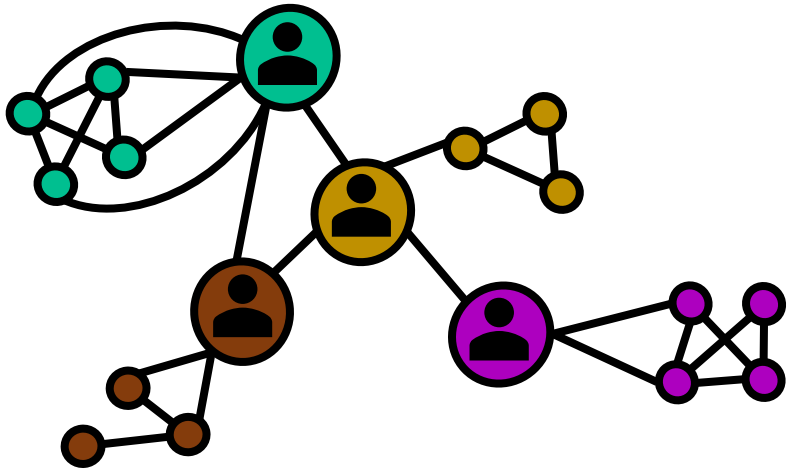
| Importance of people / links | Spread of information / virus | Communities / Polarization | Influence maximization | Social Dynamics | Prediction of properties or links |
|---|---|---|---|---|---|

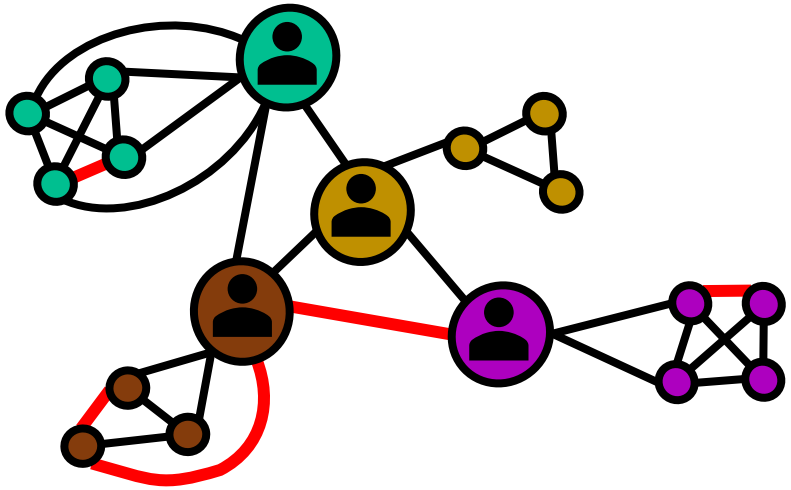
**Information Flow**

# Recommendation Systems

**Naturally**, people connect with each other by some kind of similarity [McPherson2001]

Same interests, ideas, hobbies...





Recommender Systems (RecSys) are based on the same

Create connections by user similarity based on features and connections

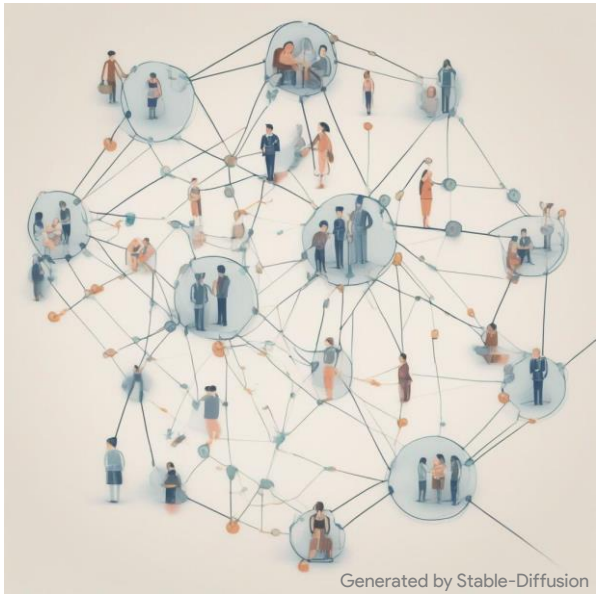
e.g.: if we share lot of frieds but we are not already connected → suggest connection

McPherson, Miller, et al. "Birds of a feather: Homophily in social networks." *Annual review of sociology* (2001)

# Challenges:

- Accurately and efficiently quantify polarization [Hohmann, 2023]
- Identify and quantify causes in the generation of the polarization [Santos, 2021]
- Identify new links to enforce diversity (destroy filter bubbles) [Masrour, 2020]
- Identify links that maximize the information flow in the network [Arnaiz-Rodriguez et al, 2024]

Hohmann, Marilena, et al. "Quantifying ideological polarization on a network using generalized Euclidean distance." Science Advances (2023)
Santos, Fernando P., et al. "Link recommendation algorithms and dynamics of polarization in online social networks." PNAS (2021)
Masrour, Farzan, et al. "Bursting the filter bubble: Fairness-aware network link prediction." AAAI (2020)
**Arnaiz-Rodriguez, Adrian, et al. "Structural Group Unfairness: Measurement and Mitigation by means of the Effective Resistance."** ***Submitted*** **(2024)**




# Structural Social Capital

## Effective Measurement and Mitigation of Information Access and Control disparities in Social Networks using Spectral Graph Theory

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# Structural Group Unfairness: Measurement and Mitigation by means of the Effective Resistance


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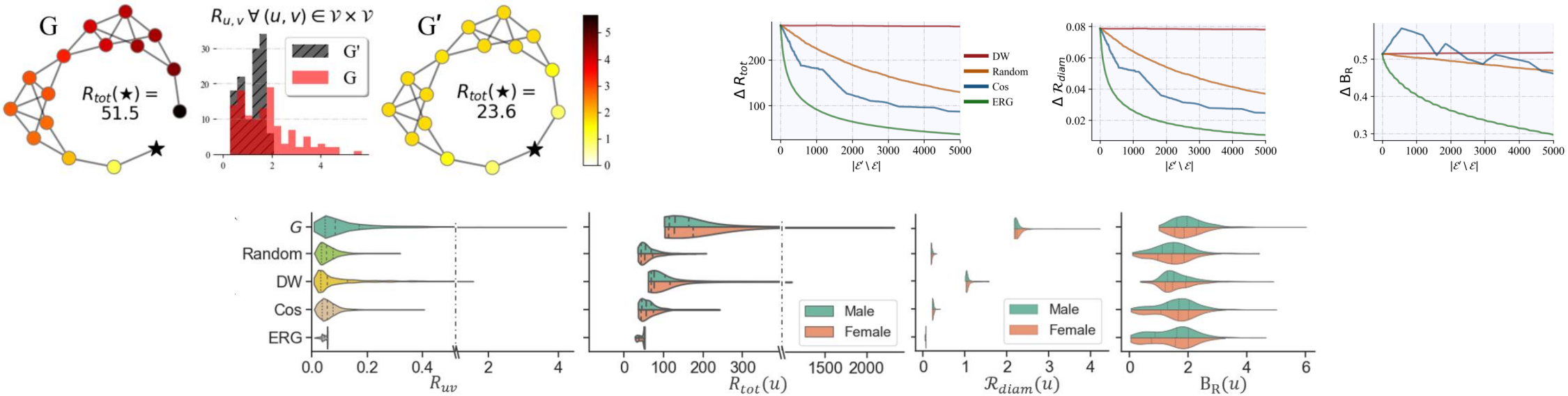


https://arxiv.org/pdf/2305.03223.pdf

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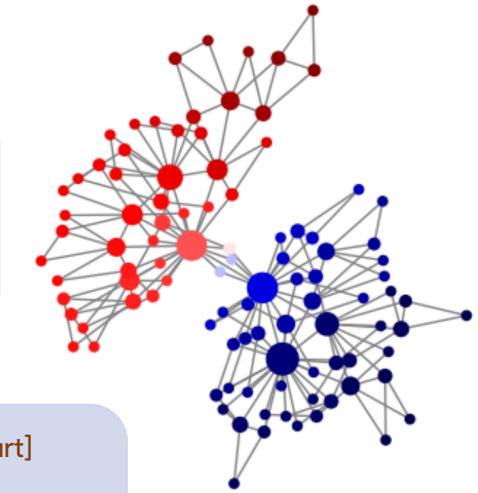
# What is Social Capital?

**Social Capital**

- Power derived from your **connections** on the network
- Networks, relationships, and **norms of trust and reciprocity** within a community or society that facilitate cooperation and collective action



**Structural Social Capital**

- Social capital derived from the **position** of a person in a network [Burt]
- **Position determines the control and access of information**

  *Job and financial opportunities, education information, health information power to spread out the word or influence others,...*

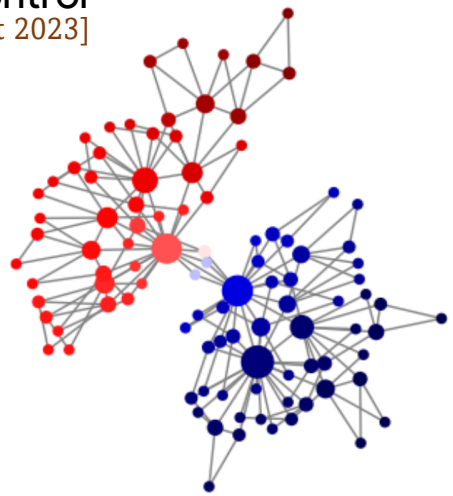
The flow of information in the network determines the social capital of individuals
Some of them will be more central and important for the flow

Source: Google

Burt, Ronald S . The network structure of social capital. *Research in organizational behavior* (2000)

# Unfairness in Social Capital

Different people have different structural role→ **Individual** disparities wrt information access and control
[Bashardoust 2023]



[Arnaiz-Rodriguez et al, 2024]

**Groups** are distributed in the network in different manners
→ different flow of information on different groups
→ different social capital (Unfairness in Social Capital)
→ **Structural Group Unfairness**

Accurate Measurement

Effective Mitigation


Burt, Ronald S . The network structure of social capital. *Research in organizational behavior* (2000)
Bashardoust, A. Reducing Access Disparities in Networks using Edge Augmentation. *FAccT* (2023)
**Arnaiz-Rodriguez, Adrian, et al. Structural Group Unfairness: Measurement and Mitigation by means of the Effective Resistance. (2024)**

# Measure Group Social Capital

**Information flow** in the network determines the **node's information access and control**

- IF usually based as **pairwise distances** between nodes

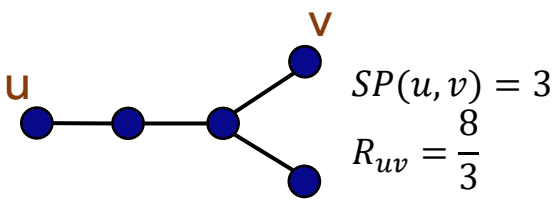
Previous work use metrics that

- **Fail to capture the global properties of the topology** → Shortest-path
- Using measures that lack theoretical guarantees → Independent Cascade

## We propose to use Effective Resistance ($R_{uv}$) as the pairwise distance metric

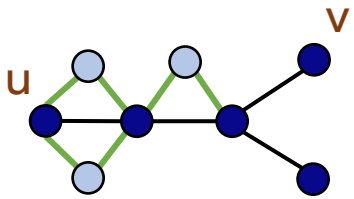
- Theoretical measure of information distance between nodes and hence of structural social capital
- Captures **global behavior** and **long-range** dependencies
- $\mathbf{R_{uv}}$ is the *expected* time to reach **v** from **u** and come back to **u**
  - i.e., average of all existing paths between u and v!
  - Captures the number and length of paths between nodes
  - The smaller ER is the more and shorter paths between u and v are

$$R_{uv} = (\mathbf{e}_u - \mathbf{e}_v)\mathbf{L}^{\dagger}(\mathbf{e}_u - \mathbf{e}_v)^{\top}$$




$SP(u,v) = 3$

$R_{uv} = \frac{8}{3}$




$SP(u,v) = 3$

$R_{uv} = \frac{11}{6} < \frac{8}{3}$

We define the (inverse) social capital of a single node as the total sum of all it's distances to everyone else

$$\mathrm{R_{tot}}(u) = \sum_{v \in \mathcal{V}} R_{uv}$$

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# Measures of Group Social Capital

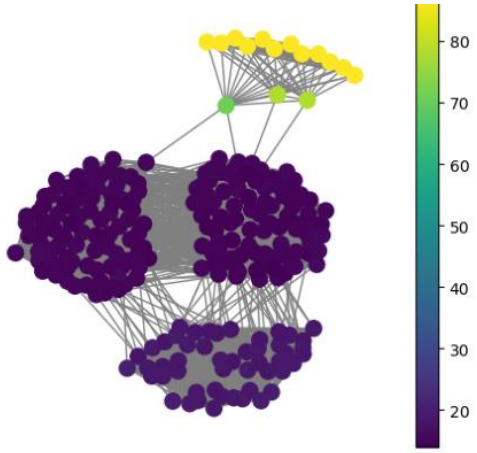
*Access*

## Group Isolation

$$\mathrm{R}_{\mathrm{tot}}(S_i) = \mathbb{E}_{u \sim S_i}[R_{tot}(u)]$$

Expected $\mathbf{R_{uv}}$ when sampling one node from group $S_i$ and another node at random

**Intuition**: average distance to everyone for the nodes of the group

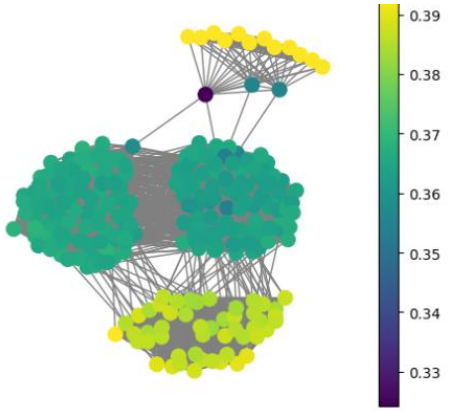



## Group Diameter

$$\mathcal{R}_{\mathrm{diam}}(S_i) = \mathbb{E}_{u \sim S_i}[\max_{v \in \mathcal{V}} R_{uv}]$$

Expected **worst-case $\mathbf{R_{uv}}$** when sampling one node from group $S_i$ and another node at random

**Intuition**: average maximum distance for the nodes of the group




*Control*

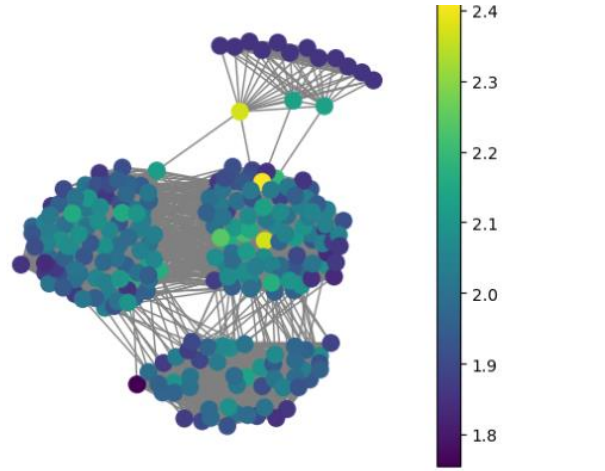
## Group Control

$$\mathrm{B}_{\mathrm{R}}(S_i) = \mathbb{E}_{u \sim S_i}[\mathrm{B}_{\mathrm{R}}(u)]$$

$$\mathrm{B}_{\mathrm{R}}(u) = \textstyle\sum_{v \in \mathcal{N}(u)} R_{uv},$$

Expected **Information bottleneck** (similar to current-flow **betweenness**)

**Intuition:** average information control for the nodes of the group

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# Measures of Structural Group Unfairness

Based on Rawlsian concept of Fairness, SGU measures the social capital gap for the most disadvantaged group

**Isolation Disparity**

$$\mathrm{R}_{\mathrm{tot}}(S_i) = \mathrm{R}_{\mathrm{tot}}(S_j),\ \forall\, i, j \in SA.$$

$$\Delta\,\mathrm{R}_{\mathrm{tot}} = \max_{i,j \in SA}(\mathrm{R}_{\mathrm{tot}}(S_i) - \mathrm{R}_{\mathrm{tot}}(S_j)).$$

**Diameter Disparity**

$$\mathcal{R}_{\mathrm{diam}}(S_i) = \mathcal{R}_{\mathrm{diam}}(S_j),\ \forall\, i, j \in SA.$$

$$\Delta\,\mathcal{R}_{\mathrm{diam}} = \max_{i,j \in SA}(\mathcal{R}_{\mathrm{diam}}(S_i) - \mathcal{R}_{\mathrm{diam}}(S_j)).$$

**Control Disparity**

$$\mathrm{B}_{\mathrm{R}}(S_i) = \mathrm{B}_{\mathrm{R}}(S_j) = 2 - \frac{2}{|\mathcal{V}|},\ \forall\, i, j \in SA.$$

$$\Delta\,\mathrm{B}_{\mathrm{R}} = \max_{i,j \in SA}(\mathrm{B}_{\mathrm{R}}(S_i) - \mathrm{B}_{\mathrm{R}}(S_j)).$$

| $G$ | $R_{tot} \downarrow$ | | $\mathcal{R}_{diam} \downarrow$ | | $\mathrm{B}_{\mathrm{R}} \uparrow$ | |
|---|---|---|---|---|---|---|
| Facebook (female) | 221.4 | $\Delta\,\mathrm{R}_{\mathrm{tot}}$ | 2.29 | $\Delta\,\mathcal{R}_{\mathrm{diam}}$ | 1.93 | $\Delta\,\mathrm{B}_{\mathrm{R}}$ |
| Facebook (male) | **179.8** | 41.62 | **2.25** | 0.042 | **2.03** | 0.107 |
| UNC28 (female) | 608.6 | $\Delta\,\mathrm{R}_{\mathrm{tot}}$ | 2.11 | $\Delta\,\mathcal{R}_{\mathrm{diam}}$ | 1.99 | $\Delta\,\mathrm{B}_{\mathrm{R}}$ |
| UNC28 (male) | **586.3** | 22.4 | 2.11 | 0.006 | **2.00** | 0.009 |
| Google+ (female) | 564.1 | $\Delta\,\mathrm{R}_{\mathrm{tot}}$ | 1.31 | $\Delta\,\mathcal{R}_{\mathrm{diam}}$ | 1.81 | $\Delta\,\mathrm{B}_{\mathrm{R}}$ |
| Google+ (male) | **287.7** | 276.4 | **1.24** | 0.078 | **2.32** | 0.51 |

*Group social capital and SGU in real-world online social networks. Group with the largest social capital is highlighted in bold.*

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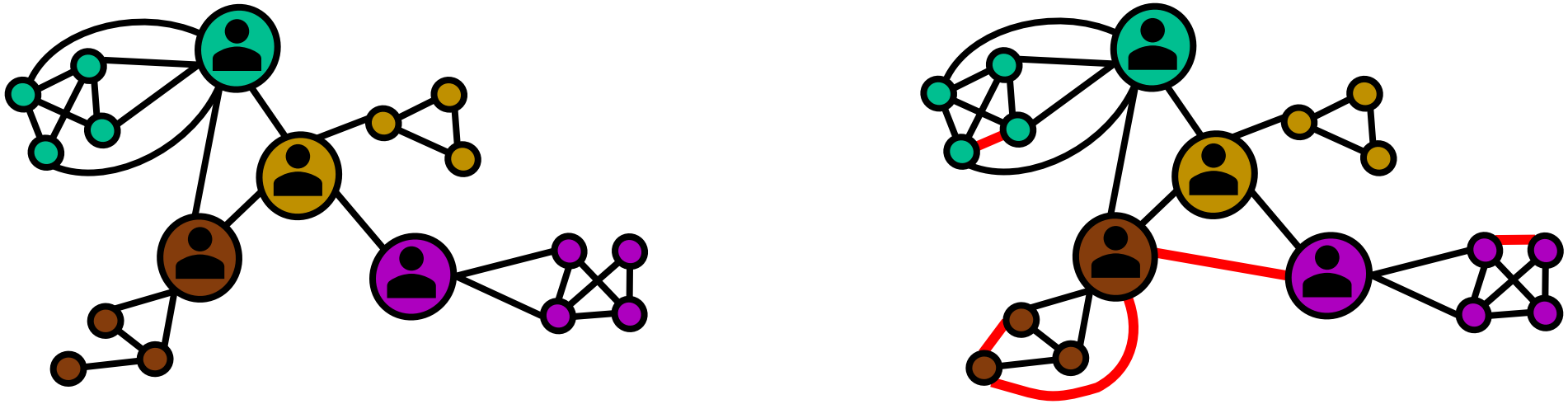
# How to mitigate SGU?

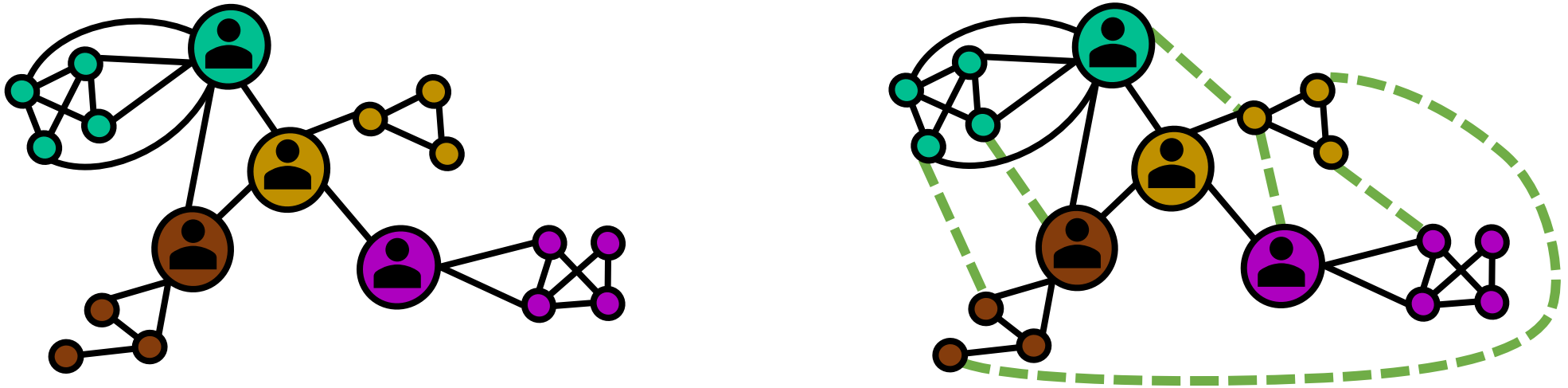


Common RecSys create connections by user similarity leading to same dynamics of segregation, polarization

RecSys do not maximize Information Flow on the graph, but **engagement → Same SGU**

# How to mitigate SGU? Weak ties

We propose a greedy Edge Augmentation Strategy based on weak ties



Create weak connections improves the network's information flow
Reduces polarization, discrimination and **isolation**
Improves diversity and **optimal flow of information**

Granovetter. The strength of weak ties: A network theory revisited. *Sociological theory* (1983)
Burt. 2004. Structural holes and good ideas. *American journal of sociology* (2004)
Arnaiz-Rodriguez, et al. Structural Group Unfairness: Measurement and Mitigation by means of the Effective Resistance. (2024)

# ERG: Edge Augmentation to mitigate SGU

- Use **Effective Resistance informed edge addition** reduce the disparities in social capital between groups and individuals **while also improve the social capital for all groups**

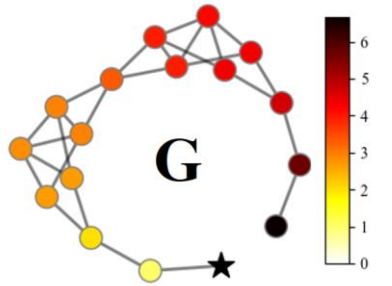
- Problem defined as a budgeted densifier problem

$$G' = \min_{G'=(\mathcal{V},\mathcal{E}')} \mathbb{E}_{u,v\sim V\times V}\left[R_{uv}\right] \quad \text{s.t.} \quad |\mathcal{E}' \setminus \mathcal{E}| = B \quad \mathcal{E} \subset \mathcal{E}'$$

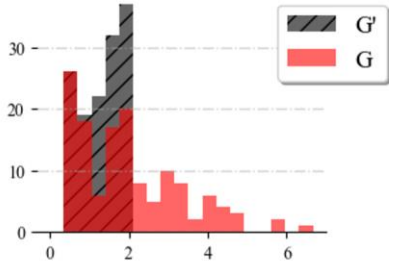
**Algorithm 1:** ERG-Link

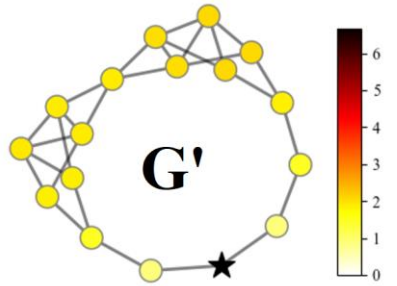
**Data:** Graph $G = (\mathcal{V}, \mathcal{E})$, a protected attribute $SA$, budget $B$ of total number of edges to add
**Result:** New Graph $G' = (\mathcal{V}', \mathcal{E}')$ with $B$ new edges

$\mathbf{L} = \mathbf{D} - \mathbf{A}$;
$S_d = \text{argmax}_{S_i \forall i \in SA} \text{R}_{\text{tot}}(S_i)$ ; // Identify the most disadvantaged group
**Repeat**
  $\mathbf{L}^\dagger = \sum_{i>0} \frac{1}{\lambda_i}\phi_i\phi_i^\top = \left(\mathbf{L} + \frac{\mathbf{1}\mathbf{1}^\top}{n}\right)^{-1} - \frac{\mathbf{1}\mathbf{1}^\top}{n}$;
  $\mathbf{R} = \mathbf{1}\,\text{diag}(\mathbf{L}^\dagger)^\top + \text{diag}(\mathbf{L}^\dagger)\mathbf{1}^\top - 2\mathbf{L}^\dagger$ ; // Compute effective resistance
  $C = \{(u,v) \mid u \in S_d \text{ or } v \in S_d, (u,v) \notin \mathcal{E}'\}$ ; // Select edge candidates
  $\mathcal{E}' = \mathcal{E}' \cup \arg\max_{(u,v)\in C} R_{uv}$ ; // Add edge with maximum effective resistance
  $\mathbf{L} = \mathbf{L} + (\mathbf{e}_u - \mathbf{e}_v)(\mathbf{e}_u - \mathbf{e}_v)^\top$; // Fast update of $\mathbf{L}$
**Until** $|\mathcal{E}' \setminus \mathcal{E}| = B$;
**return** $G'$;




$R_{u,v} \forall (u,v) \in \mathcal{V} \times \mathcal{V}$

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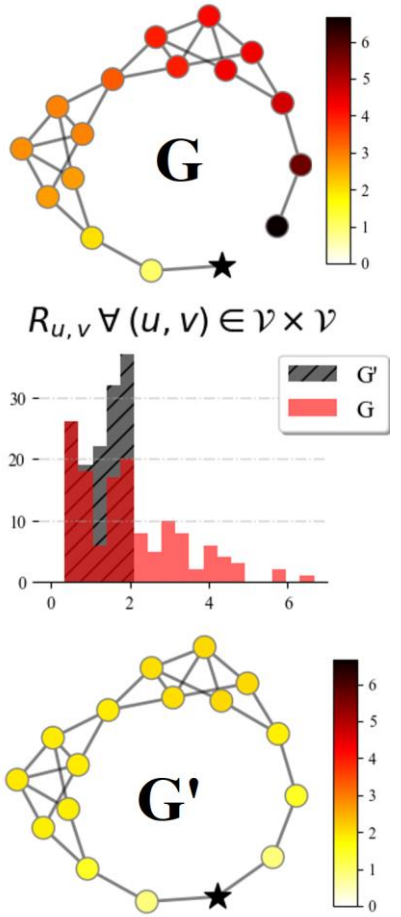
# ERG: Edge Augmentation to mitigate SGU

- Use **Effective Resistance informed edge addition** reduce the disparities in social capital between groups and individuals **while also improve the social capital for all groups**

Compute all pairwise ER distances **add** the edge with **maximum ER** that has -at least- one endpoint on the discriminated group

We have a budget of B edges to add, so we have to be very effective

As ER is heavily theoretically grounded and studied in Theoretical CS, this simple approach is effective and provides strong insights

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# Experiments on Real World: Group SC and SGU

Results on 3 real-world online social networks: Facebook, UNC and Google+

(a) Facebook ($B$=50)

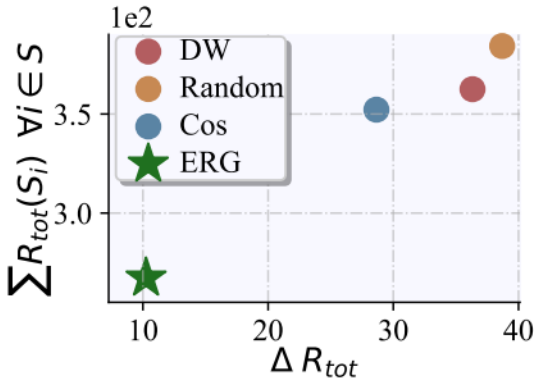
| | $\Delta$ $R_{tot}$ | $\Delta$ $\mathcal{R}_{diam}$ | $\Delta$ $B_R$ |
|---|---|---|---|
| $G$ (original) | 41.62 | 0.042 | 0.107 |
| Random | 38.7 | 0.039 | 0.108 |
| DW | 36.3 | 0.031 | 0.104 |
| Cos | 28.7 | 0.029 | 0.120 |
| ERG | **10.3** | **0.009** | **0.098** |

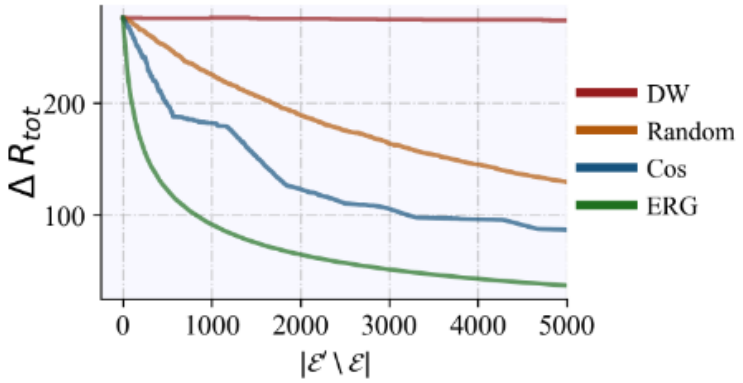
(b) UNC28 ($B$=5000)

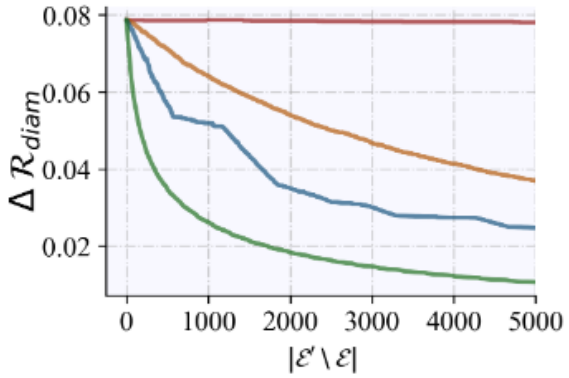
| | $\Delta$ $R_{tot}$ | $\Delta$ $\mathcal{R}_{diam}$ | $\Delta$ $B_R$ |
|---|---|---|---|
| $G$ (original) | 22.4 | 0.006 | 0.009 |
| Random | 19.8 | 0.005 | 0.014 |
| DW | 22.2 | 0.006 | 0.004 |
| Cos | 19.1 | 0.005 | 0.102 |
| ERG | **8.8** | **0.002** | **0.003** |

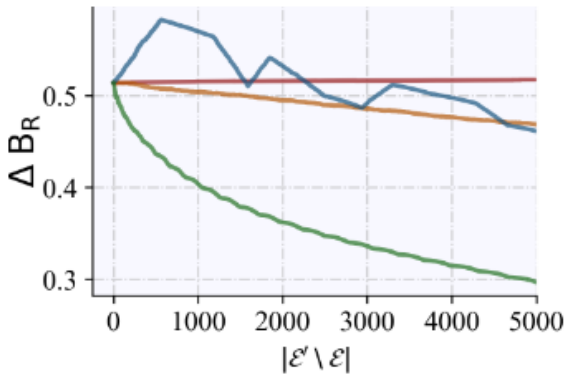
(c) Google+ ($B$=5000)

| | $\Delta$ $R_{tot}$ | $\Delta$ $\mathcal{R}_{diam}$ | $\Delta$ $B_R$ |
|---|---|---|---|
| $G$ (original) | 276.4 | 0.078 | 0.51 |
| Random | 129.4 | 0.037 | 0.47 |
| DW | 274.1 | 0.078 | 0.51 |
| Cos | 86.8 | 0.025 | 0.47 |
| ERG | **37.1** | **0.011** | **0.29** |




**Effectively mitigates the social capital gap for the most disadvantaged group while increasing the social capital of all groups**

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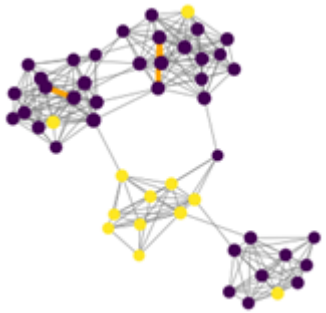
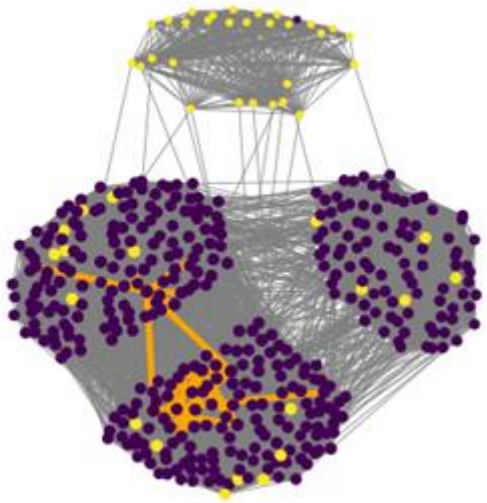
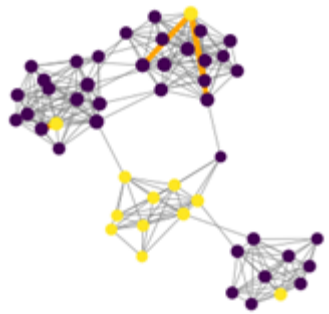
# Effect on the Network Structure

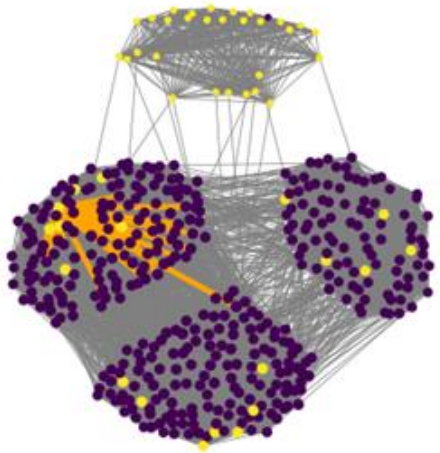
RecSys



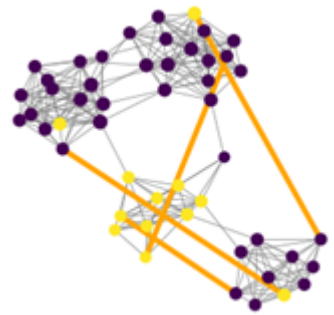


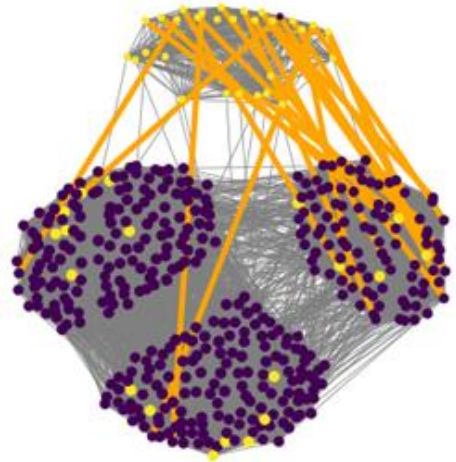
RecSys + Priorization





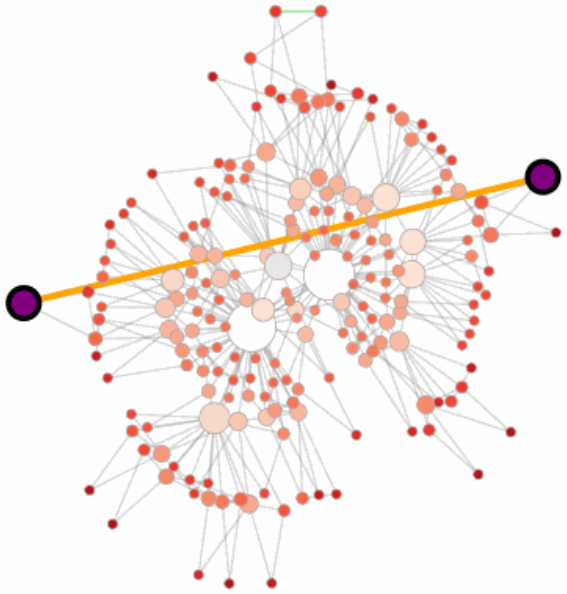
**ERG**





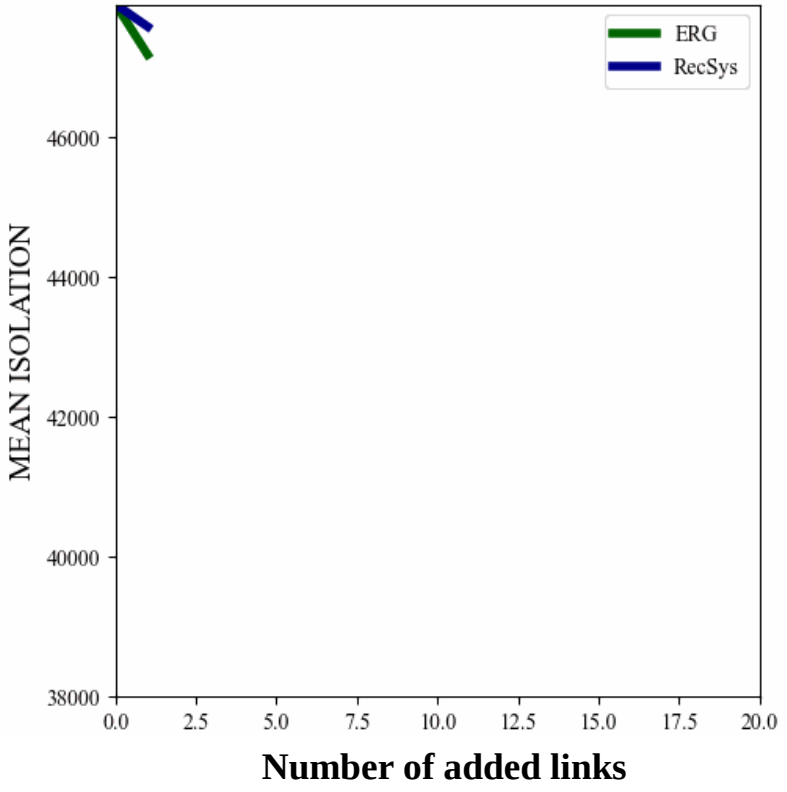
# Effect on Individual Social Capital

Intervention to maximize the information flow

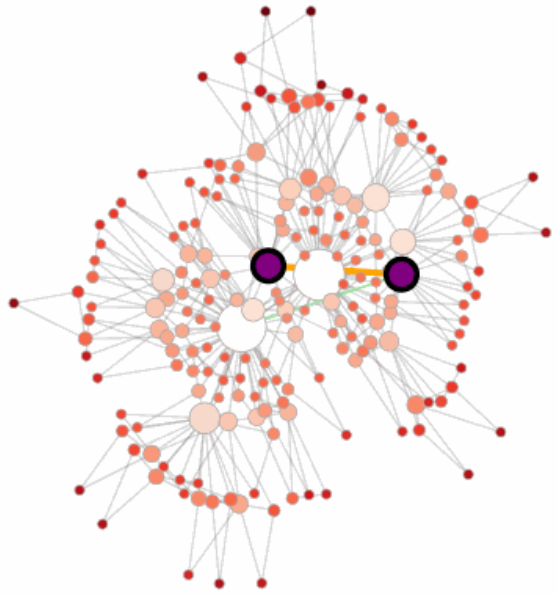


**Average Individual Isolation**

The lower the better the info is flows in the network



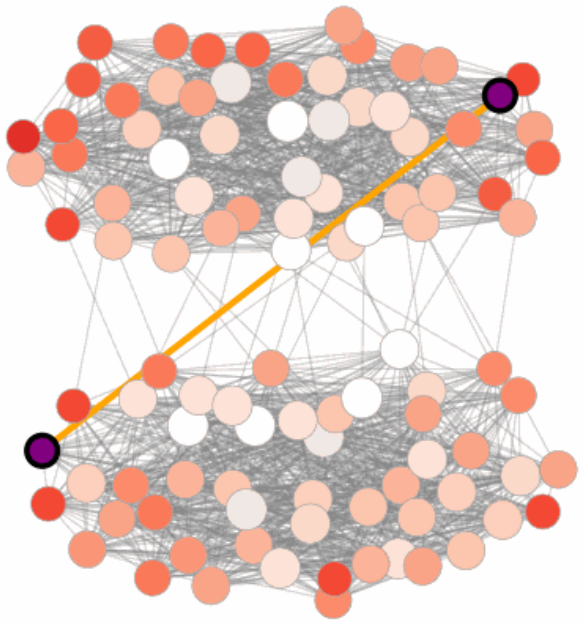

Classical RecSys



Low — Isolation of a person — High

# Effect on Individual Social Capital

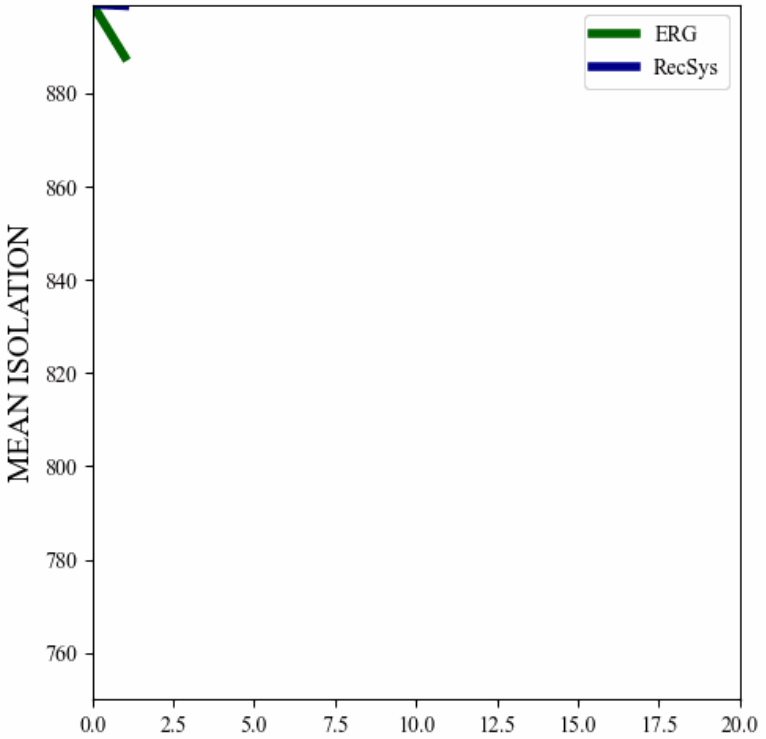
Intervention to maximize the information flow



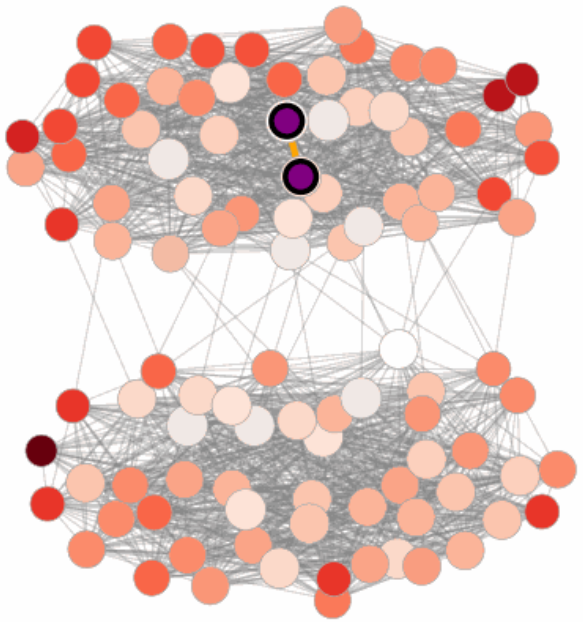
**Polarized Society**

**Average Individual Isolation**

The lower the better the info is flows in the network




Classical RecSys



Low — Isolation of a person — High

# Main takeaways

Social Networks are very useful to analyze social dynamics or make predictions about people and their relationships

Specific [Arnaiz-Rodriguez, Curto and Oliver 2024]:

Your position on a **social network** defines the access and control of information you have

Social Capital

It is not easy to **quantify it** and intricated to **align the** literature in sociology and graph theory and fairness

Quantitative measures of social capital sociology-aligned based on graph theory

**Effective resistance: isolation, diameter and control**

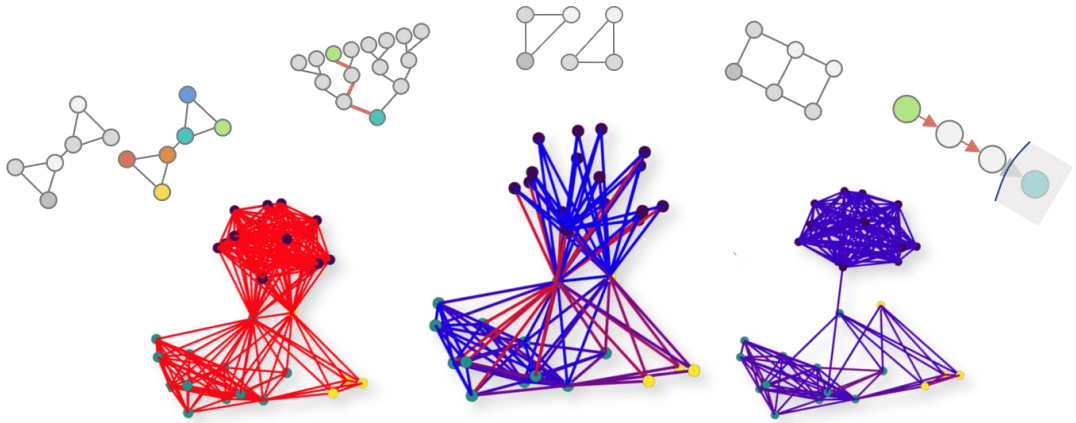
There are some individual and group inequalities and not trivial to fix

Fixed by an edge augmentation strategy based on the weak**est** links

**ERG: Adding the edge that maximizes the information flow on the whole network**

Arnaiz-Rodriguez, Adrian, Georgina Curto, and Nuria Oliver.
"Structural Group Unfairness: Measurement and Mitigation by means of the Effective Resistance." *Submitted.* (2024)

# Rewiring tutorial in LoG conference



## Graph Rewiring from Theory to Applications in Fairness

https://ellisalicante.org/tutorials/GraphRewiring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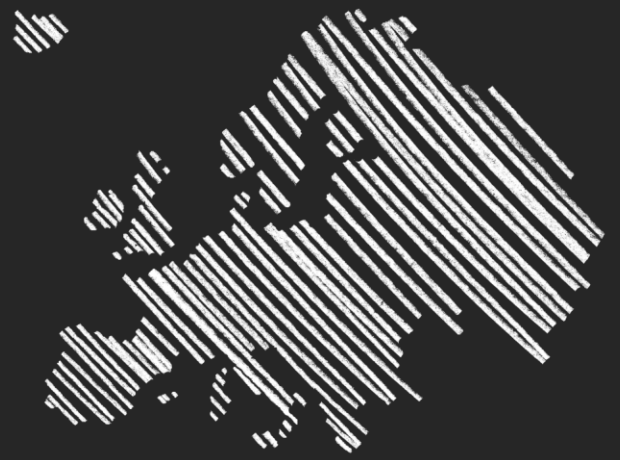
Arnaiz-Rodríguez, A. et al. "DiffWire: Inductive Graph Rewiring via the Lovász Bound" Learning on Graphs Conference (2022)
Arnaiz-Rodríguez, A. et al. “Tutorial on Graph Rewiring: From Theory to Applications in Algorithmic Fairness” Learning on Graphs Conference (2022)



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